

Enhancing Bregmanised NLTV Image Denoising using Savitzky Golay filter and SVD

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- **Abstract**— The existence of noise in digital images hinders the visual effect. To improve the visual quality a well performing denoising technique is a necessity. This paper focuses on image denoising method using Nonlocal Total Variation scheme. Nonlocal Total Variation based denoising technique helps in preserving the edges at the cost of computational load.To reduce the computational load and to speed up the process Split Bregman Optimization technique is employed to the NLTV model. To enhance this denoising Savitzky Golay filter and SVD is incorporated before applying NLTV denoising.The experimental results shows an improvement in the denoising strength which was observed using the standard quality assessment techniques such as SNR, MSE, and PSNR.

Index Terms—Edge Preservation, Image Denoising, Nonlocal Means, Savitzky Golay Filter, Split Bregman Optimization, Singular Value Decomposition, Total Variation,Image Quality Assessments.

1 INTRODUCTION

Image denoising is a technique which has been actively re-searched upon since the evolution of image processing. The image noise can be thought of as the variations in the intensity distribution and gets introduced during the image acquiring process due to the electronics involved or while getting transmitted through the channel. Most of the existing image denoising algorithms are based on assumptions on the parameters used, the type of noise etc and the method compliant for an image may not produce the same result for another sort of an image.

The competent study of an image can be done when there are elevated textures and features. This takes us to the goal of denoising algorithm that is to conserve image quality by maintaining the edges and features, avoiding over smoothing and to reproduce an ideal image. Many refined denoising techniques are available to obtain images of good quality. The period of 1940's saw the performance of Weiner filter [1] and followed many successful denoising algorithms which includes the averaging filters, anisotropic filtering, Wavelets, PDE's, TV regularization [2], [3], [4], [5].

In this paper, 2D gray scale image is preferred for experimental purpose. The gray scale image as we know has intensity distribution of 256 levels and lacks colour information. A zero represents black and intensity level 255 represents white. Most of the denoising algorithms use Gaussian noise distribution for the experimental purpose. The denoising algorithm so created can also be eminently used for other noises like Poisson, Speckle and Salt and Pepper types. This paper uses Random noise and Gaussian noise distributions.

The recent years saw a rising interest in non-linear filtering technique, the Nonlocal means which was proposed by Buades *etal* [6] in 2005. The Nonlocal means involves replacing each pixel with a weighted average of other pixels with similar neighbourhoods. Self similarity or redundancies in an image always exist and taking the average of the pixels will result in less noise as specified by Orchard *etal* [7]. Though it showed a notable result, it was computationally expensive and the edge preservation remained still a striking problem. In 2009, Xavier Bresson [9] combined Nonlocal means with Total Variation Regularization to produce resultant image with enhanced edges and other important texture features. Pixel wise as well as patch based Nonlocal approach is available in which this paper adopts a pixel based approach. The Split Bregman was concatenated along with the proposed technique to increase the SNR value as well as the computational speed and could come forward with worthy results. There has been a lot of experiments carried in the arena of Total Variation denoising and the catchiness of the algorithm lies in the fact that the higher intensities were preserved which paved a pathway of competence for wavelet based algorithms.

In this work, a satisfactory Peak Signal to Noise Ratio (PSNR) value is achieved by incorporating Savitzky Golay filter before denoising and Singular Value Decomposition (SVD) image reconstruction using limited number of components after image denoising. We have assessed the approach using the three quality estimation techniques Signal to Noise Ratio (SNR), Mean Square Error (MSE) and PSNR. Savitzky Golay is a particular type of polynomial based low pass filter which is well adapted for smoothing and helps in preserving higher intensity values. Despite the fact that significant results are produced, Savitzky Golay filtering approach has not yet been broadly pondered upon in signal filtering area. Denoising techniques requires definite measurements to be done on the denoised image to comprehend the competence of the approach being raised, for which the fine inferences can be done

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by comparing it with the original image. PSNR should be high for the resultant image indicating that the algorithm is remarkable enough to be put forth as an excellent contribution towards quality improvement of images.

The further sections of the paper are organized as follows: Section 2 gives an insight into the Savitzky Golay filter. Section 3 deals with the proposed approach followed by sub sections which enlighten upon the Nonlocal means, Total Variation, Split Bregman and Singular Value Decomposition. The paper concludes with the results and discussions in section 4 and the conclusion in section 5.

2 SAVITZKY GOLAY FILTER

The idea of the Savitzky Golay (SG) filtering technique was introduced in 1964 and gained notice with its ability to maintain sharp features in the image. The alternate filtering techniques like the moving average failed in preserving the edge features. Along with smoothing out the data, it proved successful in maintaining the minima and maxima effectively by utilizing the least squares technique [10]. The idea of SG filter was to find the filter coefficients that preserve higher moments or features in an image. In here, each point is a least square fit which is not a constant but a polynomial that is quadratic and is done by using a moving window assuming that the data points are equally spaced to eliminate the computational difficulty. Though equivalent to a smoothing operation least squares polynomial fit is efficiently performed in a one step procedure [11].

The mathematical form of the least squares fit can be explained as follows [11]:

$$g_i = \sum_{n=-n_L}^{n_R} c_n f_{i+n} \quad (1)$$

Here g_i is the estimated signal value and f_{i+n} is the input signal. If the pixel at i is taken into consideration then n_L is the number of data points towards the left and n_R is the number of data points towards its right and constant

$$c_n = \frac{1}{n_L + n_R + 1}$$

The SG filter tries to find out these filter coefficients for which the higher intensity values are preserved. The window is moved from f_{i-n_L} to f_{i+n_R} and least squares fit are performed to replace each pixel value. The SG filters are most useful for larger values of n_L and n_R where only small amount of smoothing will be done. The obtained resultant image is used for further processing. That is the smoothed out image with enhanced edge is applied to the Nonlocal TV algorithm.

3 PROPOSED APPROACH

The work explicitly focuses on the Nonlocal Total Variation with Split Bregman method to which a smoothing approach

and SVD image reconstruction is applied to increase the algorithmic strength to obtain an image with edge features and a higher PSNR rate. The Savitzky Golay filter is used to smooth out the noises present in the image with a higher degree of edge maintenance. Followed by NLTV bregmanised denoising and the prominent factor is the SVD reconstruction of image.

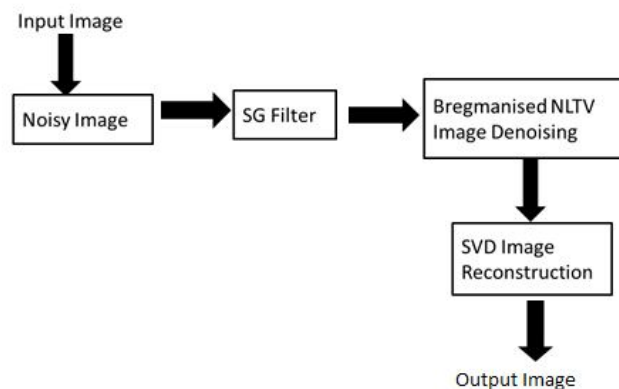


Figure 3. Diagrammatic Illustration of the Proposed Approach

3.1 NONLOCAL MEANS

The Nonlocal means algorithm was first formulated in 2005 by Buades *et al* and has found immense applications in wider area of research covering medical images to satellite images. It is a generalisation of the Neighbourhood filter originally developed by Yaroslavsky in 1985. The pixel or patches that we consider may not be nearer to the pixel under consideration. That is, any point can directly interact with any other point in the image domain and hence the name Nonlocal. The denoising of images can be done efficiently with the help of Nonlocal means algorithm which uses the redundancy present in the image. By redundancy it means the pixels having the similar intensity characteristics.

The Nonlocal means is a kind of an averaging filter in the sense that it uses the weighted average of the neighbourhood pixels and replaces the pixel value under consideration [8]. The efficiency of Nonlocal means lies in preserving the features by not causing blur or loss of edges in the denoised image. But the shortcoming of Nonlocal means lies in its computational complexity. The approach does not make any assumptions of the image other than considering that it has a lot of self similarity. Self similarity refers to those pixels with similar neighbourhood.

The observation model can be depicted as

$$v(i) = x(i) + n(i) \quad (2)$$

Where x of image size $N \times M$ is corrupted by additive white Gaussian noise or Random noise n and v is the noisy image. The Nonlocal means filtered image model can be represented by the equation given below which the standard framework is

proposed by Buades *etal*,

$$NLv(x) = \frac{\int_{\Omega} w(x,y)u_0(y)dy}{\int_{\Omega} w(x,y)dy} \quad (3)$$

Here, $u_0(y)$ is the noisy image. $NLv(x)$ computes the Non-local means of the noisy image. Whereas $w(x, y)$ is the weights assigned based on the similarity between patches located at x and y . The similarity measure is decided based on the Euclidean distance function.

The weights can be defined as,

$$w(x, y) = \exp\left(-\frac{\int_{\Omega} G_a(z)|u_0(x+z)-u_0(y+z)|^2 dz}{h^2}\right) \quad (4)$$

The value $\int_{\Omega} G_a(z)|u_0(x+z)-u_0(y+z)|^2 dz$ is the distance function which plays a role in deciding the weight between the patches at locations x and y . G_a is the Gaussian function with standard deviation a . The weight parameter ranges between zero and is positive and symmetric. Whereas, the constant h , is the weight decay control parameter or degree of filtering. The parameter plays a crucial role in denoising the image. It controls the degree of decaying of exponential function and hence the degree of decaying of the weight as a function of the Euclidean distance. If the parameter h is set too low then noise removal will be poor and if set too high the denoised image will become blurred. The size of the searching window we consider is a crucial parameter in Nonlocal means denoising approach.

For discrete points i and j for x and y belonging to Ω equation 4 can be formulated as

$$u_i = \frac{\sum_j w_{ij}u_j}{\sum_j w_{ij}} \quad (5)$$

Where u_i is the denoised image, w is the weight parameter. The window size for the work is considered as 11×11 and was not made adaptive. Similarly, all the other parameters were also kept constant which include the parameter h and the weight.

3.2 VARIATIONAL FRAMEWORK FOR NL

The Nonlocal means was adapted along with the variational architecture and it was widely studied by Gilboa and Osher, Zhou and Scholkopf [12], [13] and other eminent scholars. The Nonlocal Total Variational framework algorithmic view makes it altogether a non linear approach. The Total Variation based techniques is broadly used in inverse problems like denoising, inpainting etc which gives out excellent results.

The main principle behind Total Variation denoising is the signals with spurious details will have high total variance and hence the gradient of the image will also be high. In a way to

preserve the edge details and recreating an estimate of the original image iteratively, it does a minimization approach of the total variance. The benefit of the combination of the two techniques appears to be that the method manages to capture the best features, the patch redundancy in NL means and the nice denoising of edges in TV [14].

The classical TV minimization was proposed by Rudin *etal* [15] which is minimization of energy functional. Gilboa and Osher proposed a Nonlocal TV method as depicted below,

$$\inf_u F(u) = \int_{\Omega} |\nabla_{NL}u| + \frac{\lambda}{2}(u-u_0)^2 dx \quad (6)$$

In which the Nonlocal gradient operator is replaced as follows,

$$\int_{\Omega} \sqrt{\int_{\Omega} (u(y)-u(x))^2 w(x,y)dy} + \frac{\lambda}{2}(u-u_0)^2 dx \quad (7)$$

Where, u_0 is the noisy image and the parameter λ is the regularization factor. The regularization parameter needs to be carefully chosen. The first term is the regularization term which helps in smoothing of the image noise and the second term is the data fidelity term which helps in obtaining the original image. The iterative process of NLTV can be optimized using the convex unconstrained optimization technique which is the Split Bregman iterative procedure.

3.3 SPLIT BREGMAN NLTV MINIMIZATION

A well posed optimization procedure was proposed by Tim Goldstein and Stanley Osher [16] which can be incorporated very well with TV based methods. The mathematical derivation is based on the paper by Xavier Bresson. The fundamental idea of the Split Bregman Optimization is introducing an auxiliary variable in to the NLTV functional and the equation changes to the form depicted by Xavier Bresson as follows,

$$\inf_{u,d} = \int_{\Omega} |d| + \frac{\lambda}{2}(u-u_0)^2, \text{ such that } \nabla_{NL}u = d \quad (8)$$

Here, d is the auxiliary variable introduced to make the process computationally efficient. The equation is now a constrained minimization problem. To make it an unconstrained minimization problem, the Split Bregman iteration is to be formulated as,

$$\inf_u F(u) = \arg \min_{u,d} \int_{\Omega} d + \frac{\lambda}{2}(u-u_0)^2 + \frac{\beta}{2}|d-\nabla_{NL}u-b^k|^2 dx \quad (9)$$

The auxiliary variable b accelerates the iteration. The above mentioned equation (9) uses two sub problems one for u and the other for the auxiliary variable which is the bregman distance function d . The minimization of u can be attained using Gauss Seidal iterative scheme and the minimization of d by a

soft thresholding formula.

The two sub problems is classified as follows,

minimization of u^{k+1}

$$\lambda(u - u_0) - \beta \operatorname{div}_{NL}(d^k - \nabla_{NL}u - b^k) = 0 \quad (10)$$

minimization of d ,

$$d^{k+1} = \frac{\nabla_{NL}u^{k+1} + b^k}{|\nabla_{NL}u^{k+1} + b^k|} \max(|\nabla_{NL}u^{k+1} + b^k| - \beta, 0) \quad (11)$$

Here, the variable k represents the present iteration and $k+1$ the next iterative level. After the minimization the Bregman variable b is updated as

$$b_{ij}^{k+1} = b_{ij}^k + \sqrt{w_{ij}} (u_j^{k+1} - u_i^{k+1}) - d_{ij}^{k+1} \quad (12)$$

The splitting of the minimization problem into two sub problems makes it easy to optimize and hence the complexity in performing the computations is drastically reduced. The two images, at present iteration and at the previous iteration will be compared and iteration ends when the specified tolerance rate is reached.

The algorithmic outline can be depicted as follows:

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Initialize :  $u_0 = u, d = 0, b = 0$ 
while  $\|u^{k+1} - u^k\| \geq \text{tolerance}$ 
    solve the subproblem of  $u$ 
    solve the subproblem of  $d$ 
    update Bregman variable  $b$ 
end
    
```

3.4 IMAGE RECONSTRUCTION USING SVD

Singular Value Decomposition has been a significant area under discussion in linear algebra since its development and its finest capabilities found feasibility in digital image processing too. The transparency of the technique lies in the fact that with smaller set of values the original image can be reconstructed by preserving the most important features and textures in the image. The fundamental concept of SVD is transformation of the matrix in to three matrices. The digital image is also considered to be in the form of matrix, and henceforth an image can be reformed into the three matrices [17].

In this work, the image after denoising which is in the form of a matrix, applying SVD on it, transforms in producing two orthonormal matrices and a matrix with singular values in the diagonal. The decomposition is pictorially shown in figure 3.4.1. In that figure matrix A with m rows and n columns is decomposed into two orthonormal matrices U and V . The SVD transformation can be computed as $A = U\Sigma V^T$. The matrix

Σ 'have singular values σ_i in decreasing order. That is, with increasing rank the singular values decreases considerably. The first singular values have high intensities and the intensity level decreases as the order increases. An interpretation of it may be viewed as the image information content will be high and for the last components it will be less. The SVD image reconstruction process can be explained as follows:-

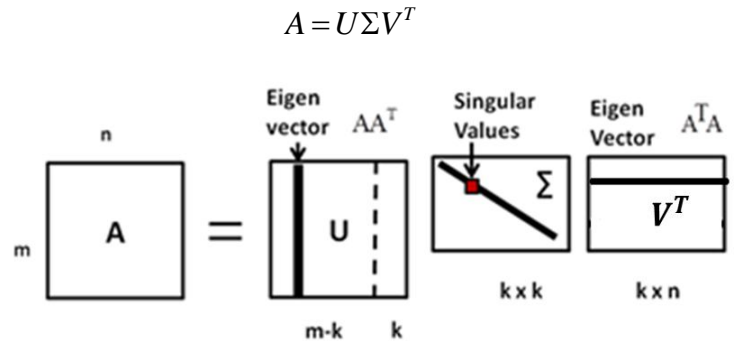


Figure .3.4.1. Illustration of SVD Decomposition

If there are k singular values then using only r values ($r < n$), we can reconstruct the image by preserving the required edge and texture image details. That is the reconstruction equation is as follows,

$$A = \sum_{i=1}^r \sigma_i u_i v_i^T \quad (13)$$

Here r singular values are used for the image reconstruction. By means of employing the SVD image reconstruction, we obtained an increase in the PSNR value which shows that the artifacts in the image are reduced to a certain extent. The experimental result is compared by taking various components range and the plots shows that there is slight change in the MSE and PSNR values which is clearly attributed in the results and discussions section of the paper.

4 QUALITY ESTIMATION TECHNIQUES

The performance of the denoising method can be quantitatively analysed using the quality metric peak signal to noise ratio and the unit of measurement is in decibels. Gaussian noise model is considered for most of the denoising algorithms and the variance of various values is added to the test image to study the performance of the proposed algorithm. The PSNR is defined as below,

$$PSNR(u_0, u) = 10 \log_{10} \frac{255^2}{MSE(u_0, u)} \quad (14)$$

Where, $MSE(u_0, u)$, is the difference between the original image and the denoised image given by the equation,

$$MSE = \frac{\sum (u - u_0)^2}{m \times n}, \text{ where } m \times n \text{ is the size of the image.}$$

5 RESULTS AND DISCUSSION

The basic idea behind the projected realization was to incorporate SVD with a filtering approach. The study showed a result with good PSNR which highlights the usability of SVD image

reconstruction approach of images. The denoising results are presented in Fig 5.1

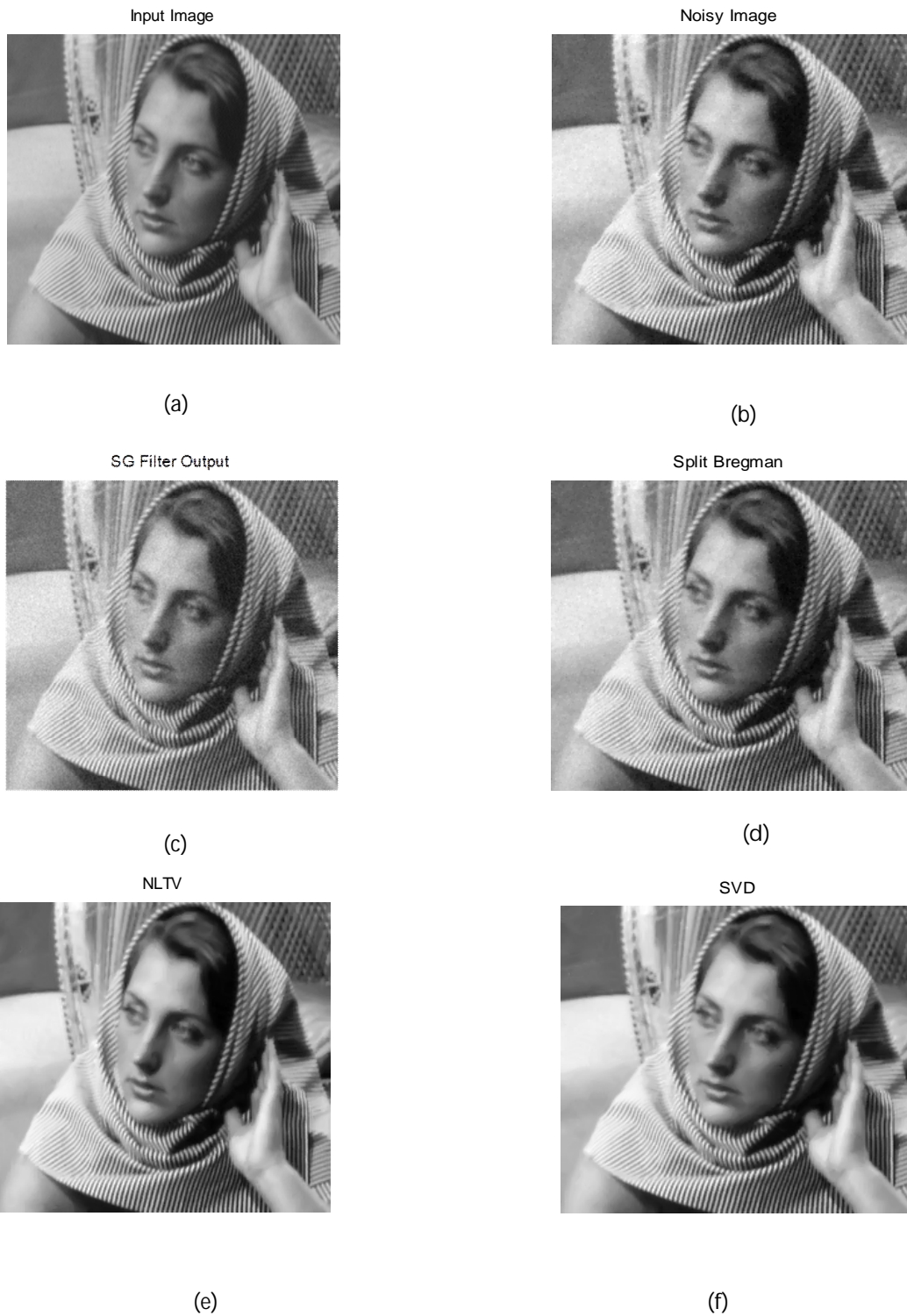
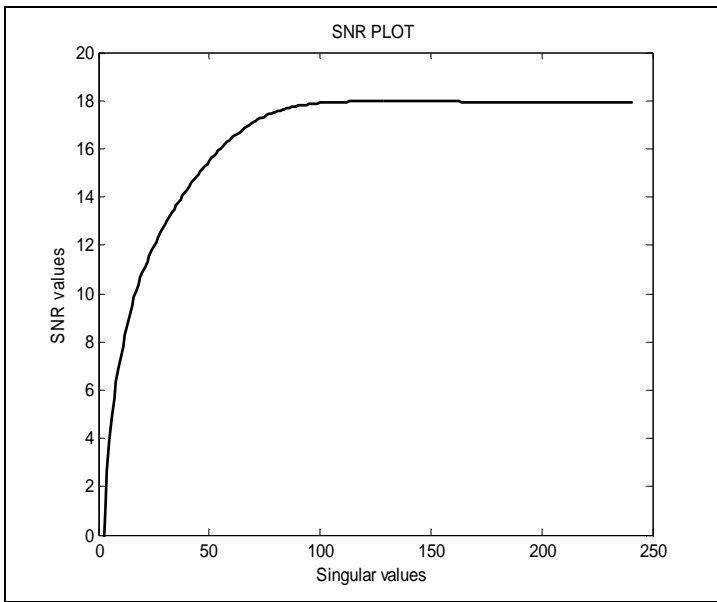
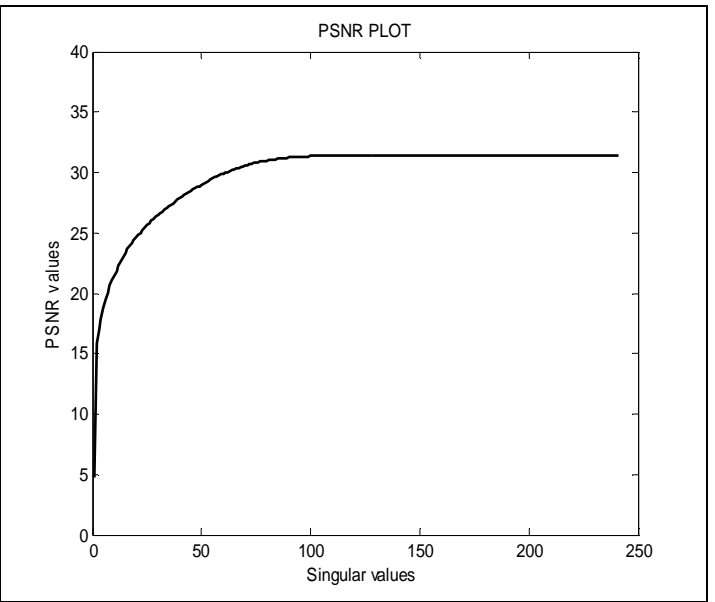


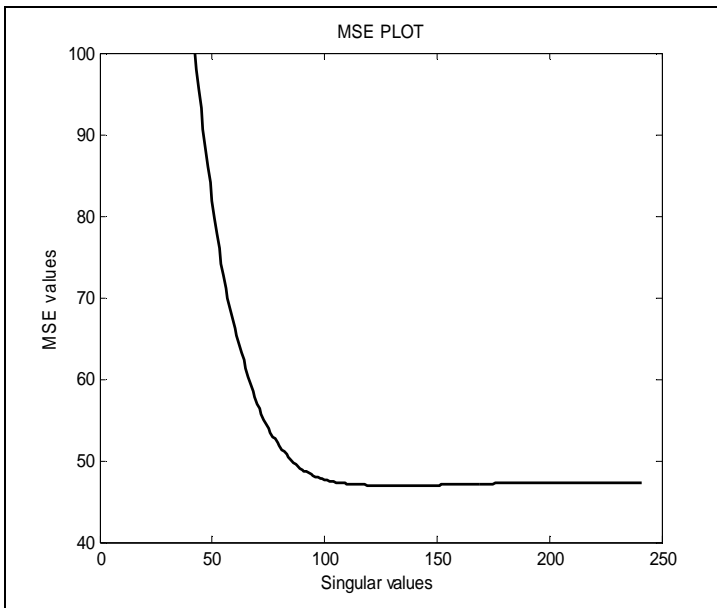
Figure 5.1.(a), (b-Random Noise), (c), (d), (e), (f) Denoising results for Barbara Image



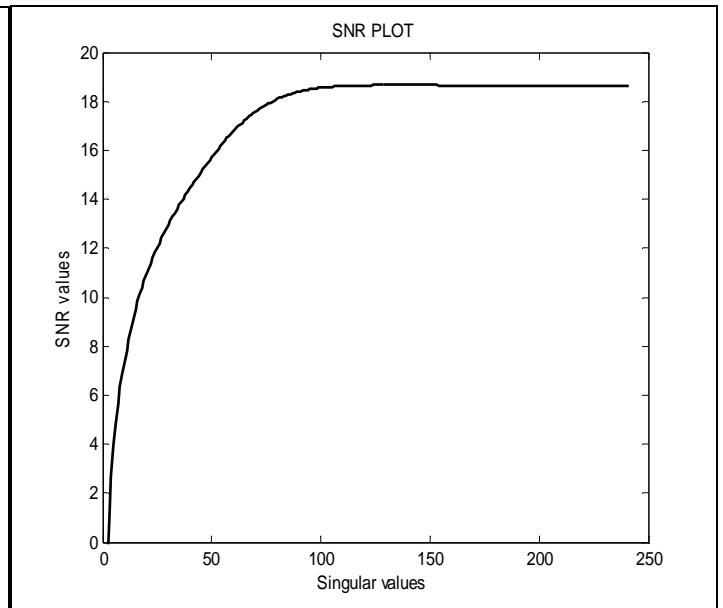
(a)



(b)



(c)



(d)

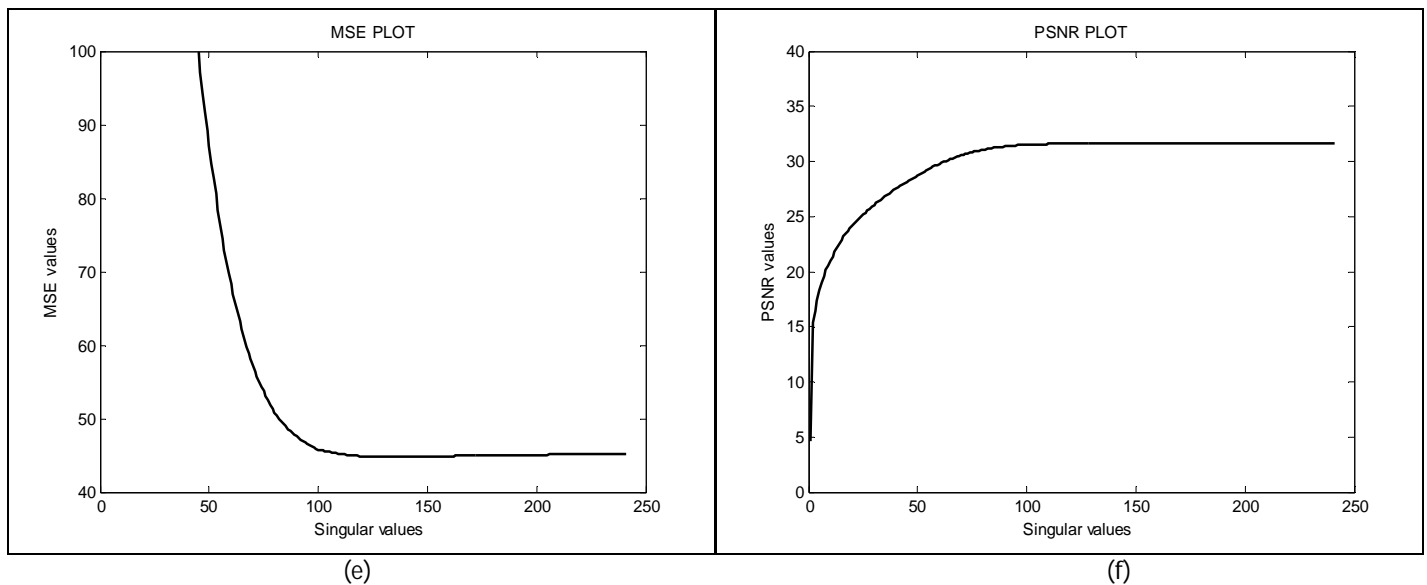


Figure 5.2. Plots shows the results for SNR, MSE, PSNR for Gaussian (a), (b), (c) and Random Noise (d), (e), (f) experiments

The SNR, MSE and PSNR values were calculated using the standard formulas using MATLAB and the outputs were plotted. For Gaussian noise a maximum PSNR of 32.042 dB was obtained whereas for Random noise a maximum PSNR of 30.83 dB was gained.

The experiment was conducted using Gaussian noise as well as Random noise and is done in MATLAB version environment. The Gaussian noise with mean zero and a variance of .001 were considered for the test. The test was performed on Barbara image with a size of 247 by 266 pixels in it. The edge features were preserved successfully in the denoised image.

It can be inferred from the Figure 5.2. (a), (c), (d), (f) that the SNR and PSNR value is maximum at a point when the singular values is between 100 to 150. The value of PSNR and SNR is minimum when only one singular value is considered. As the singular value components increases, the number of information content gradually increases, which can be observed from the quality metrics measured. An observation made from the obtained graph figure 5.2 is that there is a slight decrease in the SNR and PSNR value for higher amount of singular components. This indicates that there is a possibility of noise components at lower intensity values which do not contribute any information to the image reconstruction. Similar observations can be made for the random noise experiments as well.

6 CONCLUSION

In this paper, our work addresses the fundamental preprocessing step image denoising. We have proposed a denoising technique which uses the Savitzky Golay filter before performing the NLTV denoising with Split Bregman Optimization to preserve the higher intensities. SVD image reconstruction enhances the NLTV denoised image. The salient feature of the overall method includes: (1) smoothing is done as a default

processing by SG filter, (2) NLTV denoising helps in preserving the edge information, (3) Split Bregman Optimization in reducing the computational complexity and speeds up the process, (4) SVD image reconstruction which improves the PSNR value of the denoised image. The experiment is carried on the Barbara image and the proposed method works reasonably well with Gaussian and Random noise models.

As a future work the weighting parameter used in the proposed denoising method can be made adaptive which might further improve the denoising results.

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